I. Probability

① y : label ② x : observation (attributes)

$P(y)$: prior $P(y|x)$: likelyhood.

$P(x|y)$: posterior

③ joint probability $P(x,y) = P(x|y)P(y) = P(y|x)P(x)$

$P(x_1, x_2) = P(x_1)P(x_2)$ if x_1, x_2 are independent.

④ marginal probability $P(x) = \sum_i P(x, y_i)$

$= \sum_i P(x|y_i)P(y_i)$

II. Bayes rule.

① single observation : From I.③, we know $P(x|y)P(y) = P(y|x)P(x)$

so, $P(y_i|x) = \frac{P(x|y_i)P(y_i)}{P(x)}$

from I.④, $= \frac{P(x|y_i)P(y_i)}{\sum_i P(x|y_i)P(y_i)}$

since $P(x)$ is a fixed value.

$P(y_i|x) \propto P(x|y_i)P(y_i)$

② multiple $x_1, x_2, x_3 \cdots x_d$. Assume they are independent, then

$P(x_1, x_2, x_3 \cdots x_d | y) = P(x_1|y) \cdot P(x_2|y) \cdot \cdots \cdot P(x_d|y)$

$= \prod_{i=1}^{d} P(x_i|y)$

substitute into

To compute likelyhood $P(y_i|x)$ on $(x_1, x_2, x_3 \cdots x_d)$!

III. Tracking with Dynamics.

① Find smooth, coherent trajectory of moving targets

② current state + A model of expected motion = predicted (the distribution of) next state.

③ search around predicted area → measurement (noisy)

④ predicted next state + measurement = improved estimate

loop forever!

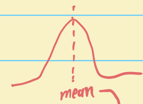
⑤ loop 可被看作 detect + predict = tracking.

Assumption: continued motion pattern (no sudden moving objects or viewpoint)

IV. Tracking as Inference.

State X: position + velocity
Measurement Y: noisy observation of underlying state.

① At time t. $X_{t-1} \to X_t$ (predict)
Y_t. (measurement)



pdf.

② goal: recover the distribution of state X_t. from:
- All observations so far.
- Model about dynamics of state transitions.

most likely position.
average guess. (Exp(X))

③ prediction: $P(X_t | Y_0, Y_1, Y_2 \ldots Y_{t-1})$ prior. without Y_t.
measurement: $P(X_t | Y_0, Y_1, Y_2 \cdots Y_t)$ posterior with Y_t.
Tracking: propagate $P(X_t | Y_t)$ over time.

④ why probability? A: You have something uncertain!
A_1: Difference between X & Y. such like include velocity in X but not in Y.
A_2: Y is noisy due to detection algorithm / device.
A_3: Y may be missing nor invalid for time t.

V. Markovian assumption.

① $P(X_t | X_0 \cdots X_{t-1}) = \underbrace{P(X_t | X_{t-1})}_{\text{dynamic model.}}$ only immediate past matters.

② $P(Y_t | X_0, Y_0, X_1, Y_1, \cdots X_t) = \underbrace{P(Y_t | X_t)}_{\text{observation model}}$ measurements dependent on current state.

$$\begin{array}{ccccccc} X_1 & \rightarrow & X_2 & \rightarrow & X_3 & \cdots \rightarrow & X_t \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ Y_1 & & Y_2 & & Y_3 & & Y_t \end{array}$$

V2. Tracking as induction.

① initial guess $P(X_0)$
② give Y_0.
③ Update $P(X_0)$
④ repeat.

Prediction

Given: $P(X_{t-1} | y_0, \ldots, y_{t-1})$

Guess: $P(X_t | y_0, \ldots, y_{t-1})$

$$= \int P(X_t, X_{t-1} | y_0, \ldots, y_{t-1}) \, dX_{t-1}$$

joint distribution

$$= \int P(X_t | X_{t-1}, y_0, \ldots, y_{t-1}) P(X_{t-1} | y_0, \ldots, y_{t-1}) \, dX_{t-1}$$

$$= \int \underbrace{\boxed{P(X_t | X_{t-1})}}_{\text{dynamics model.}} \underbrace{\boxed{P(X_{t-1} | y_0, \ldots, y_{t-1})}}_{\text{corrected estimate from } t-1.} \, dX_{t-1}$$

Independence assumption

Next state probability of getting a specific point is the integral of weighted getting the point from all points before

Correction

Given predicted value $P(X_t|y_0,\dots,y_{t-1})$ and y_t

Compute $P(X_t|y_0,\dots,y_t)$ Bayes rule.

$$=\frac{P(y_t|X_t,y_0,\dots,y_{t-1})P(X_t|y_0,\dots,y_{t-1})}{P(y_t|y_0,\dots,y_{t-1})}$$

$$=\frac{P(y_t|X_t)P(X_t|y_0,\dots,y_{t-1})}{P(y_t|y_0,\dots,y_{t-1})}$$ predicted.

$$=\frac{P(y_t|X_t)P(X_t|y_0,\dots,y_{t-1})}{\int P(y_t|X_t)P(X_t|y_0,\dots,y_{t-1})dX_t}$$

Really a normalization

observation mode → if you really at X_t, what is the likelihood of getting the measurement y_t.

$P(X_t, y_t | y_0 \cdots y_{t-1})$

$$\begin{cases} = P(X_t|y_0\cdots y_t)\cdot P(y_t|y_0\cdots y_{t-1}) \\ = P(y_t|X_t, y_0\cdots y_{t-1})\,P(X_t|y_0\cdots y_t) \end{cases}$$

$\therefore P(X_t|y_0\cdots y_t) = \dfrac{P(y_t|X_t,y_0\cdots y_{t-1})P(X_t|y_0\cdots y_t)}{P(y_t|y_0\cdots y_{t-1})}$

$\because y_t$ only depends on X_t,

$P(y_t|X_t, y_0\cdots y_{t-1}) = P(y_t|X_t)$